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Limits to Product Differentiation and Stable Market Positions

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Abstract

This paper deals with the concept of product differentiation and natural limits that can be placed on it arising from consumer search. Using a model of monopolistic competition, with consumers basing their buying decisions on known industry average price, quality and service parameters, it is shown that a Nash equilibrium, stable under perturbations, cannot have more than four strategies. To specify these possible strategies, a general model of consumer preference regarding price, quality and service is posited and explicit stable strategies are illustrated. This analysis will be particularly useful to modern market-positioning thinking--firms developing a new product for a particular target market can perform a similar analysis to get a clear idea of possible positions available and also those that are relatively stable.

Introduction

The phenomenon of product differentiation has become a way of life in the market place. The common misconception is that such differentiation occurs primarily in consumer goods. The same is also true of industrial goods, such as oil and steel, and services, such as legal and financial.

Theodore Levitt (1980) argues that there is no such thing as a commodity, which is a term used by economists in describing a product that is supposedly generic. He claims that all goods and services are differentiable--even if the generic product is identical, the offered product is differentiated. Even though the above argument does not explain why the product should be differentiated, but rather that it can be, it does imply that differences in consumers and/or producers may necessitate differentiating the product. An example of this forced differentiation is given by Kumar (1981) who looks at a single-good market model of monopolistic competition with identical producers. In his example, the nature of the demand function necessitates the formation of two district strategy groups in an economic equilibrium. Since there is only one good, it is necessarily differentiated by each group.

The question remains whether there is any limit to the extent to which a product can be differentiated. The implication in Levitt (1980) is that, potentially, there is none; "only the budget and the imagination limit the possibilities." On the other hand, one detects a move to an oligopolistic form of markets for most stable products, like automobiles, steel, toothpaste, etc. It is a rule of thumb to expect

three or four firms to control "most" of the market in many industries. This suggests that there might very well be limits to product differentiation.

In this paper I look at this question analytically, using the modeling technique used by Kumar (1981). I consider an industry in which consumers make their buying decisions based on aggregate statistics of industry strategies, i.e., consumers know the average price, average quality and service level in the industry and make their purchasing decision based on this information and some search procedure. Using the concept of Nash equilibrium, i.e., no firm has the incentive to change their strategy given the competitors' strategies, and the idea of structural stability, i.e., stability under small perturbations of the model, I show that the maximum number of groups (or strategies), that these firms can split into, is precisely four, when the number of aggregate statistics is three. The conclusion is that, given the assumption that consumers make their decisions based on a fixed number of aggregate statistics, there is definitely a limit on the extent of product differentiation.

I pursue the idea further to find out what types of market positioning are possible with respect to the price, quality and service characteristics. To do this, I impose a preference structure, which is quite general, a measurement scale on characteristics, which is specified in classes relative to the industry averages, and use the concept of efficient frontiers (or current technology) sets. With this structure, I show that stable frontiers have exactly four distinct

strategies in them and there are exactly 12 such frontiers possible. I explicitly enumerate them and discuss their features.

This analysis is particularly useful to modern market-positioning thinking--firms developing a product for a particular target market have a clear idea of the possible positions available and also those that are relatively stable.

I present the economic model of the market in Section 2 and prove the stability result concerning maximal differentiation in Section 3. Section 4 deals with market positioning results and I present the concluding remarks and future research efforts in Section 5.

Section 2

The Model

My model of monopolistic competition consists of n firms, each of which produce a single product for the market. Each firm's product can be differentiated, but it remains a close substitute for the products of the other firms. I will assume that the consumers of this product group base their buying decisions on the distribution of strategies of all firms. It is usual, in the literature on consumer search, to assume that this distribution is explicitly known. I deviate here by contending that consumers know only a few statistics of this distribution. For instance, it is more common for the consumers to know the average price than the entire price distribution itself. More specifically, I will assume that the statistics used in the buying decision are average price ($\overline{P}$), average quality ($\overline{Q}$) and average price service level ($\overline{S}$).

From the producer's point of view, each firm's demand function, and hence its profit function, is dependent not only on its own available strategies but also the triplet $(\overline{P}, \overline{Q}, \overline{S})$. I will assume that firm i's available strategic variables are its price (P_i), quality (Q_i) and service level (S_i). Then, let

$$\pi(P_i, Q_i, S_i, \overline{P}, \overline{Q}, \overline{S})$$

be the payoff function common to all the firms, as required by the assumption of identical firms. It is clearly symmetric in its arguments and it will be assumed that π is continuously differentiable in all its variable but not necessarily quasi-concave in its strategic variables (P_i, Q_i, S_i). As is usual, I will assume that the firms are profit maximizers over their strategic variables (P_i, Q_i, S_i) while taking the aggregate statistics $(\overline{P}, \overline{Q}, \overline{S})$ as given.

The appropriate equilibrium concept in this model is the Cournot-Nash equilibrium and I will be looking at the symmetric equilibrium since, a priori, all firms are identical. By my convention, the equilibrium exhibits product differentiation if it is in mixed strategies and the number of strategies in this mixed equilibrium is the extent of product differentiation. For instance, if there is only one strategy in the equilibrium, then there is no product differentiation since all firms are behaving alike; if there are three strategies in the mixed equilibrium, then the firms split into three groups displaying distinct strategies and necessarily differentiated products. I will assume that there are a finite number of strategies in the mixed equilibrium.

<u>Def</u>: A symmetric mixed strategy Nash equilibrium with a finite support of dimension k is given by $((P_i, Q_i, S_i, \omega_i)_{i=1}^k, (\overline{P}, \overline{Q}, \overline{S}))$ satisfying

$$\sum_{i=1}^{k} \omega_i \pi(P_i, Q_i, S_i, \overline{P}, \overline{Q}, \overline{S}) \geq \sum_{i=1}^{k} \hat{\omega}_i \pi(P_i, Q_i, S_i, \overline{P}, \overline{Q}, \overline{S}),$$

$$\omega \varepsilon \Omega, \ (P_i, Q_i, S_i) \ \varepsilon \ R(\overline{P}, \overline{Q}, \overline{S})$$

$$\text{and} \quad \sum_{i=1}^{k} \omega_i P_i = \overline{P}, \quad \sum_{i=1}^{k} \omega_i Q_i = \overline{Q}, \quad \sum_{i=1}^{k} \omega_i S_i = \overline{S}$$

$$\text{for all } \hat{\omega} \varepsilon \Omega$$

where $R(\overline{P}, \overline{Q}, \overline{S})$ is the set of (P_i, Q_i, S_i) which globally maximize profits at $(\overline{P}, \overline{Q}, \overline{S})$ and Ω is the set of all discrete probability measures with support of dimension k.

The above definition implies that there are k strategies $(P_i, Q_i, S_i)_{i=1}^k$ which are the best response strategies given $(\overline{P}, \overline{Q}, \overline{S})$ and that each of these strategies has a corresponding probability ω_i associated with it. This probability $(\omega_i)_{i=1}^k$ is the one that gives the highest expected profits compared to all other probability measures and also satisfies the consistency condition that the expected or average value of the best response strategy (P_i, Q_i, S_i) is indeed $(\overline{P}, \overline{Q}, \overline{S})$.

It is quite easy to show that if $((P_i, Q_i, S_i, \omega_i)_{i=1}^k, (\overline{P}, \overline{Q}, \overline{S}))$ is a symmetric mixed equilibrium, then the profit level associated with each response, that has a positive probability associated with it, must be the same (Owen 1968). We will assume that all the best

responses have a positive probability associated with them, i.e., the equilibrium is regular.

We are now ready to discuss the issue of how large k, the extent of product differentiation, can be.

Section 3

Limits to Product Differentiation

A priori, one has no reason to believe that there is a bound on the extent of product differentiation for the model presented. I am going to impose the condition of stability in the equilibrium structure and with this restriction, a definite limit on product differentiation emerges.

The concept of stability of a mixed strategy equilibrium used here is similar to that of structural stability in differential topology (Guillemin and Pollack 1974). The fundamental building block in my model is the profit function and in practice, one can estimate it to within some non-zero margin of error. Given these small perturbations in the profit function, one can ask the question whether the equilibrium in these perturbed models is similar and close to that in the unperturbed model. If so, the equilibrium in the unperturbed model is said to be stable.

Let us consider a mixed equilibrium with support of dimension k $((P_i^*, Q_i^*, S_i^*, \omega_i^*)_{i=1}^k, (\overline{P}^*, \overline{Q}^*, \overline{S}^*))$ for the model characterized by the profit function π. This implies that around a small neighborhood of $(\overline{P}^*, \overline{Q}^*, \overline{S}^*)$, there exists k local maxima and by the assumption of regularity, they are isolated. Also, the profit levels at each of these local maxima, at $(\overline{P}^*, \overline{Q}^*, \overline{S}^*)$, is the same, i.e.,

let $x_i \equiv (P_i, Q_i, S_i)$, $i=1, \ldots, k$

and $y \equiv (\overline{P}, \overline{Q}, \overline{S})$

and $D\colon R^3 \to R^{k-1}$

be the profit difference map defined by:

$$D(y) = (\pi(x_i(y),y) - \pi((x_{i+1}(y),y), \quad x_j \in S(y) \quad j=1, \ldots, k \quad i=1, \ldots, k-1)$$

where $S(y)$ is the set of local maxima at the point $y = (\overline{P}, \overline{Q}, \overline{S})$. Then, in a small neighborhood of $y^* = (\overline{P}^*, \overline{Q}^*, \overline{S}^*)$, $D(y)$ is well defined and $D(y^*) = 0$.

I will consider all perturbations π_λ of the function π such that the functional value of π and the first and second derivatives of π_λ can be made as close as needed to those corresponding values of π by choosing λ small enough. To ensure the stability of the equilibrium using profit function π, I need to show the existence of a similar equilibrium, close by and unique in a neighborhood of the original equilibrium, for all perturbations π_λ, with λ in a some open neighborhood of the value 0. (I am assuming $\pi_0 = \pi$). Three conditions have to be satisfied for such an existence, namely

(1) the local maxima sets $S(y)$ and $S_\lambda(y)$ must be close to each other in a neighborhood of y^* and the cardinality of $S_\lambda(y)$ should be equal to k.

(2) if condition (1) is satisfied, then the profit difference function D_λ is well defined in this neighborhood of y^*. Then, we need to

show the existence of $y^*_\lambda \equiv (\overline{P}^*_\lambda, \overline{Q}^*_\lambda, \overline{S}^*_\lambda)$ close to y^*, in the neighborhood were D_λ is defined such that $D_\lambda(y^*_\lambda) = 0$.

(3) Then we need to ensure the existence of a probability vector ω^*_λ such that the expected value of the best response strategy $(P^*_{i\lambda}, Q^*_{i\lambda}, S^*_{i\lambda})$ is indeed $(\overline{P}^*_\lambda, \overline{Q}^*_\lambda, \overline{S}^*_\lambda)$.

Condition (1) can be shown to be true through an application of the implicit function theorem and using the fact that the local maxima are non-degenerate critical points (Guillemin and Pollack 1974). It is the second condition which leads us the limitation on the value that k can take.

Instability Theorem: If the dimension of the aggregate statistics space $(\overline{P}, \overline{Q}, \overline{S})$ is three, then no mixed equilibrium with a finite support of dimension more than four can be structurally stable.

Proof: Note that the profit difference function

$$D: \; R^3 \to R^{k-1}$$

satisfies $D(y^*) = 0$. When $k > 4$, the dimension of the domain space is less than that of the range space. This implies that the range of D is a closed set of measure zero in R^{k-1}. Then any small perturbation of D will never hit the range of D almost surely, implying that there exists for λ, however small, a perturbation D_λ such that

$$D_\lambda(y_\lambda) \neq 0 \text{ for all } y_\lambda.$$

Then condition (2) is not satisfied and hence the equilibrium is unstable. $\square$

The result implies that if there are three statistics involved ($\overline{P}$, $\overline{Q}$, $\overline{S}$) then the firms canot split i to more than four groups, each group differentiating the product uniquely, in a stable equilibrium. Notice that it does not assure us a stable equilibrium with exactly four groups but rather that it could exist. Also, with three aggregate statistics, we could very well have a product differentiated stable equilibrium with only two groups of firms. Also, one can presumably construct an example, with more than four groups in equilibrium, in this model; but any perturbation of the example would destroy the structure of this equilibrium.

Similar dimensional argument has been used, with inverted logic, in the field of multi-dimensional scaling. One of the questions, that multi-dimensional scaling can generate answers for, is: "Given a respondent's similarity judgements about all pairs of products in a specific product-differentiated market, how many dimensions underlie this respondent's judgements about the similarity-dissimilarity of the products?" In our context, this query concerns itself with the identification of the number of aggregate statistics used in the model, given that there are n firms or groups of firms in the industry. It is well known (Churchill 1976, pp. 233-241) that we can plot these n groups somewhat arbitrarily in any (n-1) statistic space but not constraint-free in a lower dimensional space. This is similar to my generalized theorem (Kumar 1981) which states that if there are (n-1) statistics, then, at most, n groups can occur in a stable equilibrium.

Let me now return to the model with three statistics ($\overline{P}$, $\overline{Q}$, $\overline{S}$). In a stable equilibrium, my result states that there can be, at most,

four groups with differentiated strategies. What are these four possible strategies that can occur in a maximally differentiated equilibrium? I take up this question in the next section.

Section 4

Simple Domination and Market Positioning

In the previous section, using an economic market model of monopolistic competition with three aggregate statistics ($\overline{P}$, $\overline{Q}$, $\overline{S}$), I showed that there can be, at most, four groups of firms with differentiated strategies. To specify what those strategies are is beyond the scope of the preceding analysis since an equilibrium strategy would necessarily depend on consumer preferences and the cost structure of the profit-maximizing firm; in other words, without the assumption of a specific profit map !, which is functionally explicit, one cannot delve deeper into the structure of the equilibrium.

In Kumar (1981), a specific model of linear demand in price-advertising space is considered along with a single aggregate statistic, namely average price. The effect of advertising, in that model, is that of own-price information, i.e., advertising a price lower than the average industry price increases demand level and advertising a price higher than the average industry price decreases demand level. In this case, it was shown that the two specific strategies in equilibrium were:

(1) price below industry average and advertise at a positive level.

(2) price above industry average and do not advertise.

The drawback of such an analysis is that the specific functional forms of the demand and profit function are questionable.

In this section, I am going to take an intermediate course--place some reasonable assumptions on consumer preferences while not completely specifying them. I will also use the concept of efficient frontier (or current technology) sets to identify possible equilibrium strategy sets.

The assumption on consumer tastes, and therefore demand function, is that there is monotonic increasing preference over quality and service level and monotonic decreasing preference over price level. This is a fairly reasonable assumption that one can justify since it does not impose any comparative preference structure over the three characteristics. The other assumption on these characteristics is that of measurement: each of the characteristics' level will be measured on an ordinal scale, relative to the average industry characteristic level. For example, measurement of a firm's pricing strategy could be one of three classes: below average, average, above average. In fact, I will restrict the number of classes to three for each of the characteristics. A rationale for this assumption is that, since consumers are making their purchasing decisions based on average levels, they would tend to categorize any individual firm's strategy relative to the averages. For example, a consumer, using search strategy, might classify a firm as being average price, above average quality and below average service. The restriction to three classes is for ease of exposition and analysis.

The next assumption is that the active strategies in equilibrium must be those on the efficient or current technology frontier. Essentially this implies that the frontier strategies are not dominated by others on the frontier while, at the same time, the frontier dominates every other strategy not in it. Let me make these concepts more precise.

Def: A strategy for a firm is a triplet (P, Q, S), where the value of Q and S is measured as one of three classes: (1) below average, (2) average, and (3) above average. The characteristic P is measured as one of (1) above average, (2) average, and (3) below average.

e.g., (3, 1, 3) is a strategy denoting below average price, below average quality and above average service.

The above definition gives every firm a point to choose from a cube, with each lattice point referring to a unique strategy. Given the assumption on consumer preferences, the ideal strategy that consumers prefer is (3, 3, 3), namely below average price, above average quality and service level. Obviously, some of these strategies are dominated by others and cannot coexist in an equilibrium.

Def: Strategy i, namely (P_i, Q_i, S_i) is said to be superior to strategy j, namely (P_j, Q_j, S_j) if

$$P_i \leq P_j,\ Q_i \geq Q_j,\ S_i \geq S_j$$

with, at least, one strict inequality. In this case, strategy j is said to be inferior to strategy i.

<u>Def</u>: Two strategies are said to be mutually compatible if neither one is superior (or inferior).

<u>Def</u>: An efficient or current technology frontier F is a set of strategies $\{(P_1, Q_1, S_1), \ldots, (P_k, Q_k, S_k)\}$ such that

(a) each strategy in F is mutually compatible with every other strategy in F.

(b) all the other strategies not in F are either superior or inferior to some strategy in F.

(c) to preserve the notion of average, each characteristic must be represented in, at least, the above and below average levels in the set F.

Condition (a) represents the idea that only those strategies, which are preference-comparable as regarded by the consumer, can survive in an equilibrium. Condition (b) ensures that all the other available strategies for the firm are dominated either in a superior or inferior sense. Since we have assumed that the measurement of the characteristics are relative to the industry average, the strategies in F have to be consistent with the notion of industry average. For example, strategy (3, 3, 3) satisfies (a) and (b) but cannot be considered as a valid frontier since this is the only choice, by the industry, and hence is the industry average also--this implies that it cannot be below and above the industry average!

The rationale behind the set F being a proxy for the Nash equilibrium set is that, since the strategies in F are compatible, it is conceivable that consumer demand is distributed amongst them in such a way as to equalize the profits obtained by each strategy in F.

Since the strategic space is finite (in fact, 27 possible strategies) and the frontier is clearly defined, the efficient frontiers can be completely enumerated. This was done using the computer and the entire list appears in the Appendix. The following result emerges from analyzing this list:

Result 1: There are 113 possible efficient frontier sets. Of these, 6 of them have cardinality 3, 40 of them have cardinality 4, 57 of them have cardinality 5, 9 of them have cardinality 6 and 1 of cardinality 7.

The instability in Section 3 stipulates that only those with cardinality of, at most, 4 can exist in a stable equilibrium. Then, we have to reconcile the above result which allows cardinalities of 5, 6 and 7 to be possible equilibria. The bridge between the two lies in the concept of stability and its application to the model in this section.

Result 1 lists all possible frontiers without any regard to the stability of the set. By stability of the set, I mean the structural stability with respect to small perturbations in either preferences, demand function or cost structures.

I will now describe a condition which is unstable with respect to perturbations. It will also be shown that all of the sets with cardinality of 3, 5, 6 and 7 have this condition and so also some of the sets with cardinality 4. It will then be shown that the remaining stable frontiers have cardinalities of 4, which is consistent with the instability theorem.

<u>Condition FF</u>: Consider an efficient frontier F containing two strategies (P_1, Q_1, S_1) and (P_2, Q_2, S_2). Then set F is said to exhibit condition flip-flop (FF) if any of the following holds:

(a) $P_1 = P_2$, $Q_1 = S_2$, $S_1 = Q_2$.

(b) $Q_1 = Q_2$, $P_1 = S_2$, $S_1 = P_2$.

(c) $S_1 = S_2$, $P_1 = Q_2$, $Q_1 = P_2$.

<u>Result 2</u>: Any efficient frontier which exhibits condition FF is structurally unstable.

<u>Proof</u>: Condition FF imposes a symmetry amongst two or more characteristics. Let us consider, without loss of generality, a set of F containing the two strategies (m, m_1, m_2) and (m, m_2, m_1) corresponding to (a) in condition FF. Since these are in the efficient frontier and are equilibrium strategies, the profits associated with either of these strategies must be the same. Consider the perturbation of the profit function which breaks the symmetry by favoring that strategy which has higher quality characteristic. Without loss of generality, let us assume that $m_1 > m_2$. Then (m, m_1, m_2) has higher profits than (m, m_2, m_1) and hence is superior to it. Thus, the efficient frontier F is unstable with respect to this perturbation. $\square$

Let us consider an example of the above result. Consider $F = \{(1, 1, 3), (2, 3, 2), (3, 1, 2), (3, 2, 1)\}$ and notice that F exhibits condition FF since (3, 1, 2) and (3, 2, 1) correspond to situation (a). Since F is a possible equilibrium, the profits corresponding to (3, 1, 2) and (3, 2, 1) must be the same and this

implies that the firm is indifferent between (3, 1, 2) and (3, 2, 1). Now consider a perturbation of the profit function which breaks this indifference by assigning more profits to (3, 2, 1) since it is a higher level of the quality characteristic. Then, the profit equalization is broken and F becomes unstable with respect to small perturbations. On the other hand, consider F = {(1, 1, 3), (2, 3, 2), (3, 1, 2), (3, 3, 1)}. There is no underlying symmetry, regarding the characteristics, that can be inferred from F and hence, no small perturbation of the preferences over characteristics can achieve dissolution of F through incompatibility.

Results 1 and 2 lead to the main conclusion of this section:

Stable Frontier Theorem: Given the assumptions on consumers monotonic preferences over characteristics, the measurement of the characteristics, and characterization of efficient frontier, the only stable efficient frontiers are:

(1) {(1, 1, 3), (1, 2, 2), (3, 1, 2), (3, 3, 1)}

(2) {(1, 1, 3), (1, 3, 2), (2, 1, 2), (3, 3, 1)}

(3) {(1, 1, 3), (1, 3, 2), (3, 2, 2), (3, 3, 1)}

(4) {(1, 1, 3), (2, 3, 2), (3, 1, 2), (3, 3, 1)}

(5) {(1, 2, 2), (1, 3, 1), (3, 1, 3), (3, 2, 1)}

(6) {(1, 2, 3), (1, 3, 1), (2, 2, 1), (3, 1, 3)}

(7) {(1, 2, 3), (1, 3, 1), (3, 1, 3), (3, 2, 2)}

(8) {(1, 3, 1), (2, 2, 3), (3, 1, 3), (3, 2, 1)}

(9) {(1, 3, 3), (2, 1, 2), (2, 3, 1), (3, 1, 1)}

(10) {(1, 3, 3), (2, 1, 3), (2, 2, 1), (3, 1, 1)}

(11) $\{(1, 3, 3), (2, 1, 3), (2, 3, 2), (3, 1, 1)\}$

(12) $\{(1, 3, 3), (2, 2, 3), (2, 3, 1), (3, 1, 1)\}$

It should be noted that in spite of the instability theorem allowing stable equilibria to possess four or less strategies, the stable frontier theorem dictates exactly four strategies. The strength of this result is due to condition (c) in the definition of an efficient frontier and also the measurement scale of the characteristic. If the strategies available to the firms were continuous in the characteristics rather than in discrete classes and the measurement independent of the industry average, then single strategy undifferentiated efficient frontiers such as low price, high quality and high service could very well be possible, subject to technological constraints.

The stable frontier theorem has a few features worth mentioning. Considering all the efficient frontiers, there is always one characteristic which is present only in upper and lower classes (i.e., above and below average) while the other two appear in all three classes. Also, strategy (1, 3, 3), which is the high price, quality and service strategy, is always accompanied by strategy (3, 1, 1), which is the lowest price, quality and service strategy. One would expect (2, 2, 2) or the average price, quality and service strategy to coexist with the preceding two but is not found. This is due to the observation that if (2, 2, 2) were an equilibrium strategy, then there exists an undifferentiated single strategy equilibrium, namely (2, 2, 2).

Some other cautionary statements about the stable frontier theorem is that there is no preferences shown across the twelve frontiers;

therefore, one cannot foretell which equilibrium may obtain. In fact, my own hypothesis is that one can trace a sequence of efficient frontiers (defined sans condition (c)) over the life cycle of the industry (which is the reason for denoting them also as current technology sets). As specified in the discussion after the instability theorem, one could very well experience a 5, 6 or 7-strategy equilibrium, but this will not be stable given small perturbations or shocks to the system.

Conclusion

This research is an attempt to explain the oligopolistic structure of many stable industries, such as automobiles, steel, toothpaste, refrigerators, etc. In this paper, I have considered a simple market model of monopolistic competition. The structure and concepts used are similar to a large body of literature in economics regarding product differentiation except in the assumption that I make on available information. I assume that consumers are not aware of the firm's strategy distributions explicitly, but rather some aggregate statistics of these distributions. In the model posited, I use average price, quality and service as the key distributional parameters known to everyone.

Using structural stability concepts, the instability of any product differentiated equilibrium with more than four strategies is proven. To further pinpoint what these strategies in equilibrium may be, I postulate a preference and measurement structure on the monopolistic competition model. It is then shown, using the concepts of efficient frontiers and stability, that exactly four strategies are possible on

an efficient frontier and the 12 such frontiers are listed in the stable frontier theorem.

An important extension of this research is in the direction of relaxing the number of aggregate statistics used by imposing a restriction that a few of the characteristics are more used than the others, e.g., price, quality and service constitute 90 percent of the information used. A natural result, that is yet a hypothesis, is that four strategies dominate (say, 80%) the market equilibrium, thus giving rise to an essentially oligopolistic market structure.

References

Churchill, G. A., Jr. (1976), Marketing Research Methodological Foundations, Dryden Press, pp. 233-241.

Guillemin, V. and A. Pollack (1974), Differential Topology, Englewood Cliffs, New Jersey, Prentice-Hall, Inc.

Kumar, K. Ravi (1981), "Product Differentiated Equilibrium: Structural Stability, Maximal Dispersion and Realization," unpublished Ph.D. dissertation, Northwestern University.

Levitt, Theodore (1980), "Marketing Success Through Differentiation--Of Anything," Harvard Business Review, Jan.-Feb., pp. 83-91.

Owen, G. (1968), Game Theory, W. B. Saunders Company.

Appendix

In this appendix, the problem formulation for the efficient frontier model is explained and also the listing of the efficient frontiers is given. Also specified are those frontiers which exhibit condition flip-flop (FF) and hence are not inherently stable with respect to small perturbations of the model. Those frontiers, which are not afflicted with this condition, are stable and all of these have cardinality 4.

The problem formulation is as follows:

$$\text{Strategy Space } C \equiv \{(i,j,k)\,|\,i=1,2,3,\ j=1,2,3,\ k=1,2,3\}$$

$$\text{Strategy} = (i,j,k) \equiv (P,Q,S)$$

$$\text{where price } P = \begin{cases} 1, & \text{above average} \\ 2, & \text{average} \\ 3, & \text{below average} \end{cases}$$

$$\begin{array}{l}\text{and quality } Q \\ \text{(or) service } S\end{array} = \begin{cases} 1, & \text{below average} \\ 2, & \text{average} \\ 3, & \text{above average} \end{cases}$$

$$\text{Enumerate } F = \{(i_1,j_1,k_1),\ \ldots,\ (i_m,j_m,k_m)\,|\,\text{they form an efficient frontier}\}.$$

There are 113 such efficient frontiers and these are listed in Table 1. Of these, 12 are stable and are denoted by * and the rest exhibit condition FF, which makes them unstable under perturbation.

Table 1

Number	Cardinality	Efficient Frontier Strategies					Type
1	3	113	131	322			FF
2	3	113	232	311			FF
3	3	122	313	331			FF
4	3	131	223	311			FF
5	3	133	212	331			FF
6	3	133	221	313			FF
7	4	113	122	212	331		FF
8	4	113	122	312	331		*
9	4	113	131	222	311		FF
10	4	113	132	212	331		*
11	4	113	132	221	312		FF
12	4	113	132	231	322		FF
13	4	113	132	322	331		*
14	4	113	232	312	321		FF
15	4	113	232	312	331		*
16	4	113	232	322	331		FF
17	4	122	131	221	313		FF
18	4	122	131	313	321		*
19	4	122	213	231	311		FF
20	4	122	213	312	331		FF
21	4	122	231	313	321		FF
22	4	123	131	212	321		FF
23	4	123	131	213	322		FF
24	4	123	131	221	313		*
25	4	123	131	313	322		*
26	4	123	132	212	331		FF
27	4	123	132	221	313		FF
28	4	123	213	232	311		FF
29	4	123	232	313	321		FF
30	4	131	223	312	321		FF
31	4	131	223	313	321		*
32	4	131	223	313	322		FF
33	4	132	223	231	311		FF
34	4	132	223	312	331		FF
35	4	133	212	221	311		FF
36	4	133	212	231	311		*
37	4	133	212	231	321		FF
38	4	133	213	221	311		*
39	4	133	213	221	312		FF
40	4	133	213	231	322		FF
41	4	133	213	232	311		*
42	4	133	222	313	331		FF
43	4	133	223	231	311		*
44	4	133	223	232	311		FF
45	4	133	223	313	332		FF
46	4	133	232	323	331		FF
47	5	113	122	131	212	321	FF

Table 1 (cont'd.)

Number	Cardinality	Efficient Frontier Strategies					Type
48	5	113	122	131	221	312	FF
49	5	113	122	131	312	321	FF
50	5	113	122	212	231	311	FF
51	5	113	122	212	231	321	FF
52	5	113	122	231	321	321	FF
53	5	113	131	222	312	312	FF
54	5	113	132	212	221	311	FF
55	5	113	132	212	231	311	FF
56	5	113	132	212	231	321	FF
57	5	113	132	222	231	311	FF
58	5	113	132	222	312	331	FF
59	5	113	122	131	212	321	FF
60	5	113	122	131	221	312	FF
61	5	113	122	131	312	321	FF
62	5	113	122	212	231	311	FF
63	5	113	122	212	231	321	FF
64	5	113	122	231	312	321	FF
65	5	122	131	213	221	311	FF
66	5	122	131	213	221	312	FF
67	5	122	131	213	312	321	FF
68	5	122	213	231	312	321	FF
69	5	123	131	212	221	311	FF
70	5	123	131	213	221	311	FF
71	5	123	131	213	221	312	FF
72	5	123	131	213	222	311	FF
73	5	123	131	222	313	321	FF
74	5	123	132	212	221	311	FF
75	5	123	132	212	231	311	FF
76	5	123	132	212	231	321	FF
77	5	123	132	213	221	311	FF
78	5	123	132	213	221	312	FF
79	5	123	132	213	231	322	FF
80	5	123	132	213	322	331	FF
81	5	123	132	222	313	331	FF
82	5	123	132	231	313	322	FF
83	5	123	132	313	322	331	FF
84	5	123	213	232	312	321	FF
85	5	123	213	232	312	331	FF
86	5	123	213	232	322	331	FF
87	5	132	232	313	322	331	FF
88	5	132	223	231	312	321	FF
89	5	132	223	231	313	321	FF
90	5	132	223	231	313	322	FF
91	5	132	223	313	322	331	FF
92	5	133	213	222	231	311	FF
93	5	133	213	222	312	331	FF
94	5	133	213	232	312	321	FF

Table 1 (cont'd.)

Number	Cardinality	Efficient Frontier Strategies							Type
95	5	133	213	232	312	331			FF
96	5	133	213	232	322	331			FF
97	5	133	222	231	313	321			FF
98	5	133	223	231	312	321			FF
99	5	133	223	231	313	321			FF
100	5	133	223	231	313	322			FF
101	5	133	223	232	312	321			FF
102	5	133	223	232	312	331			FF
103	5	133	223	232	313	321			FF
104	6	113	122	131	212	221	311		FF
105	6	113	132	222	231	312	321		FF
106	6	113	122	131	212	221	311		FF
107	6	123	131	213	222	312	321		FF
108	6	123	132	213	222	231	311		FF
109	6	123	132	213	222	312	331		FF
110	6	123	132	222	231	313	321		FF
111	6	133	213	222	231	312	321		FF
112	6	133	223	232	313	322	331		FF
113	7	123	132	213	222	231	312	321	FF